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MULTI-OBJECTIVE PROGRAMMING UNDER UNCERTAINTY TO SELECT PORTFOLIO: A CASE STUDY IN STOCK EXCHANGE MARKET OF IRAN

Abstract. *Recently, the economic crisis has increased uncertainty degree of person's investment in financial markets. Therefore, considering uncertainty conditions to optimize financial portfolios is very importance. This paper presents a decision support model called chance constrained goal attainment programming (CCGAP) to optimize multi-objective portfolio decision problem under uncertainty environment. CCGAP is a combination of the well known classical approach of chance constrained programming (CCP) and A-priori multi-objective approach of goal attainment programming (GAP) that is known to be an extension of goal programming (GP). The proposed model is illustrated in a real problem of multi-objective portfolio selection by Iran stock market, where goal values of stochastic objectives and rate of return of securities are considered random and normally distributed in different scenarios.*

Keywords: *Stochastic programming, Multi-objective programming, Chance constrained goal attainment programming, Portfolio selection.*

JELClassification: C61, C63, D81, G11

1. Introduction

The Markowitz covariance model (Markowitz, 1952), the classical and basically approach to portfolio optimization, is based on two conflicting optimization objectives: the risk and the expected return. The "mean-variance" methodology (Markowitz, 1952) for portfolio selection problem has been central to research activity and has served as a basis for the development of modern financial theory. Yet, the Markowitz model disadvantages include:

(i) The Markowitz model was generally criticized as not efficient with axiomatic models of preferences for choice under risk (Bell et al., 1988).
(ii) The Markowitz model was a quadratic model which finally was non-linear. Because his model was non-linear, so obtained results were often local optimum.
(iii) The mean-variance model of Markowitz was the time needed to compute the covariance matrix from historical data especially when problems were large scale and the difficulty of solving the large scale quadratic programming problems. In the literature, some algorithms such as those proposed by Sharpe (1967), Elton et al. (1976), Konno (1990), and Young (1998) are generated in order to linearize and improve the efficiency calculation of the Markowitz covariance model (Shing and Nagasawa, 1999). Serban et al. (2011) presented description of efficient frontier for a portfolio made of three assets. The originality of their work was in the combination of classification theory and risk estimation theory to determine the best assets. Amiri et al. (2011) proposed a nadir compromise programming (NCP) for optimization of multi-objective portfolio problem in Iran stock market. Georgescu (2011) presented stylized facts displayed by the Bucharest stock exchange BET index and then provided an application of GARCH modeling approach to predicting BET index mean-return and volatility-return processes.

In real world, using Markowitz model would not have desired performance when parameters are estimated. Because data about events in the past may not be known exactly due to errors in measuring or difficulties in sampling, whilst data about events in the future may simply not be known with certainty. For example, the expected return of a portfolio was used as an approximation because returns are random, and it is hardly possible that the investor can group a portfolio by attending to all of its possible returns (Liu, 1999). In many situations there is a need to make, hopefully an optimal, decision under uncertainty.

Stochastic programming deals with a class of optimization models and algorithms in which some of the data may be subject to significant uncertainty. Recently, a few authors studied stochastic portfolio selection problems. Several techniques have been introduced to solve stochastic programming problems. The most popular technique is the chance constrained programming (CCP) developed by Charnes and Cooper (1963).

The CCP method is a deterministic equivalent formulation of a stochastic problem (Charnes and Cooper, 1963). This technique allows the uncertainty related to several parameters of the problem such as the constraints coefficients, the objectives coefficients and the goals values. The main idea behind the CCP technique it to allow the decision-maker to generate the most satisfactory solutions by making compromises between the various achievement degrees of the objectives and the risk associated with these objectives. In fact, the CCP attempts to maximize the expected value of the objectives while assuring a certain probability of realization of the different constraints (Aouni et al., 2005).

Using multi-objective stochastic models for stochastic portfolio selection problems has been considered by many researches and DMs. For example, in model proposed by Shing and Nagasawa (1999) the mean and variance of return of securities have several scenarios with known probabilities. Muhlemann et al. (1978) developed a multi-objective stochastic linear programming formulation of portfolio selection problem under uncertainty.

Ballestero (2001) proposed a formulation of stochastic goal programming based on utility function and ‘‘Mean-Variance’’ model. Tang et al. (2001) proposed a chance-constrained problem of portfolio selection to choose a portfolio with minimized standard deviation under the condition that the probability that the portfolio's rate of return is greater than an expected rate of return is no less than a confidence level. Ballestero (2005) minimized portfolio semi variance as the objective function in stochastic programming model subject to standard parametric constraints which led to the mean-semi variance efficient frontier. Canakoglu and Ozekici (2009) considered this problem in a multiple period setting where the investor maximizes the expected utility of the terminal wealth in a stochastic market. Xu and Zhang (2012) considered a stochastic programming model where the objective function was the variance of a random function and the constraint function was the expected value of the random function. Tamiz et al. (1996) proposed a two-stage goal programming model for portfolio selection. Aouni et al. (2005) explicitly introduced the DM’s preferences and adapted CCP for the SGP model. They illustrated their formulation through a portfolio selection example where the goal values associated with each objective are considered normally distributed.

The GP model was proposed by Charnes et al. (1955). In the GP there is no clear relation between the weights and the solution found as you might end up in new vertices in both the solution space, as well as in the utopian set. Hence it seems appropriate to use a method that besides having advantages of GP method, can apply DM’s ideas in optimization process of stochastic problem.

The goal attainment programming (GAP) (Gembicki, 1974) is of methods in *priori category* that can be considered as a special kind of the GP method. Compared to the GP method, the GAP method has the following advantages:

(1) The GAP has fewer variables. If a multi-objective problem has K objective functions, m constraints and n variables, then compared to GAP method, the GP method will has $2K-1$ more variables, so that by increasing the dimensions of problem, this difference would be understood easier, (2) When dimensions of multi-objective

problems are increased, the GAP method has higher speed for optimizing such problems. If a linear programming problem has *n* variables and *m* constraints, then maximum number of needed iterations for optimizing this problem will be equal to $C_m^n = n!/m!(n-m)!$ (Taha, 1976), (3) The GAP method allows optimization of non-linear multi-objective problems. Gembicki and Haimes (1975) showed that the GAP can optimize non-linear multi-objective problems too. This shows capability of simultaneous optimization of linear and non-linear multi-objective problems by the GAP method, and (4) There is a specific relation between objectives preference weights and optimal values of objectives in the GAP method (Andersson, 2000) (see Section 2 for more details).

Therefore our main motive in this paper is to combine CCP approach and GAP method and present chance constrained goal attainment programming (CCGAP) method which can be used for optimizing multi-objective stochastic problems. Beside the GAP method's advantages, ideas of DM can be used in CCGAP method for decision making under uncertainty.

In order to illustrate the proposed model, we test it by a multi-objective problem of portfolio selection from the Iran stock exchange market. In this paper, in addition to parameters of rate of return, goal values of stochastic objectives are considered random too in stochastic problem of multi-objective portfolio selection.

The rest of the paper is organized as follows: In Section 2 we introduce the GAP method and explain the stochastic programming and the CCP approach. In Section 3, we propose a chance constrained goal attainment programming (CCGAP) model, which combine the GAP model and the CCP approach. The CCGAP allows DM to consider several conflicting objectives with random parameters. In Section 4, a real case is given to illustrate the CCGAP model about problem of multi-objective portfolio selection. Meanwhile we conclude this paper in Section 5.

2. Portfolio selection problem

Consider a multi-objective problem of portfolio selection as follows:

$$
\begin{aligned}
&\max \quad f_k : \sum\nolimits_{j=1}^{n} c_{kj} x_j, \quad k = 1,\dots,K \\
&\min \quad f_r : \sum\nolimits_{j=1}^{n} c_{rj} x_j, \quad r = K+1,\dots,R \\
&\text{subject to:} \\
&\sum\nolimits_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1,\dots,m \\
&\sum\nolimits_{j=1}^{n} x_j = 1, \\
&x_j \ge 0, \quad j = 1,\dots,n,
\end{aligned}
\tag{1}
$$

where f_k $k = 1,\dots, K$ are positive objectives such as liquidity, return, f_r $r = K+1, \dots, R$ are negative objectives such as risk, cost and $\sum_{j=1}^{n} a_{ij} x_j \le b_i,\ i = 1,\dots,m$ are constraints of the problem. The x_j is the decision variable of proportion to be invested in the security j and the sum of the proportions invested in securities is equal to 1.

Until now, different methods have been introduced for solving multi-objective problems, e.g. Program (1). One of these methods is GAP (Gembicki, 1974) in which, a vector of decision making goals $g = (g_1,\dots, g_t)$ is associated with a set of objectives $f = (f_1,\dots, f_t)$. The relative degree of under-or over-achievement of the goals is controlled by a vector of preference weights of objectives $w = (w_1, \dots, w_t)$. In the case that the more of the objective is better, the final model of GAP can be obtained as follows:

$$\begin{aligned}
&\min \quad y \\
&\text{subject to:} \\
&f_k + w_k y \ge g_k, \qquad k = 1,\dots,K \\
&\sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1,\dots,m \\
&\sum_{j=1}^{n} x_j = 1, \\
&x_j \ge 0, y \ge 0, \qquad j = 1,\dots,n,
\end{aligned} \tag{2}$$

where $w_k > 0$ (for $k = 1, \dots, K$) and $\sum_{k=1}^{K} w_k = 1$. The term $w_k y$ introduces an element of surplus into Program (2), which otherwise imposes that the goals be rigidly met. In Program (2), g_k is goal related to objective k.

If the case that less of the objective is better, the final model of GAP can be obtained as follows:

$$\begin{aligned}
&\min \quad y \\
&\text{subject to:} \\
&f_r - w_r y \le g_r, \qquad r = K+1,\dots,R \\
&\sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1,\dots,m \\
&\sum_{j=1}^{n} x_j = 1, \\
&x_j \ge 0, y \ge 0, \qquad j = 1,\dots,n.
\end{aligned} \tag{3}$$

The term $w_r y$ introduces an element of slackness into Program (3), which otherwise imposes that the goals be rigidly met. The weighting vector (w) enables the DM to express a measure of the relative trade-offs between the objectives. For instance, setting the vector w equal to the initial goals indicates that the same percentage under- or over-attainment of the goals (g) is achieved. The GAP method provides a convenient intuitive interpretation of the programming problem which is solvable using standard optimization procedures.

One of advantages of the GAP method is existence of a specific relation between preference weights and optimal values of each objective (Andersson, 2000). See Figure 1 for better understanding.

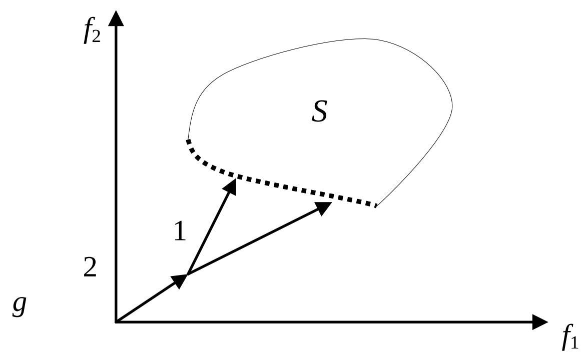

Figure 1. Representation of relation between w_k and f_k^* (for k = 1, 2) values in the GAP method

Figure 1 shows minimization of a two objectives problem in solution space S. The dotted region shows the optimal and feasible Pareto frontier of solution space S. In order to optimize this problem by the GAP method, the continued vector g shows goal values vector of each objective which are specified by DM. Also the vectors (1) and (2) show two different vectors of w_k values. In vector (1): $w_1 < w_2$ and in vector (2): $w_1 > w_2$. On this basis considering vectors (1) and (2), it can be said $f_1^* < f_2^*$ and $f_1^* > f_2^*$, respectively. For example, when value of w_k decrease, value of f_1^* will decrease too. This property allows DM to select a proper preference weight vector for decision making in order to get more desired results in minimum possible time.

In a real case, DMs do not have exact and complete information on decision objectives and constraints. For portfolio-selection problems, the collected data does not behave crisply and they are typically uncertain in their nature. Hence stochastic problem related to Program (1) can be stated as follows:

$$
\begin{aligned}
&\max \quad \tilde{f}_k : \sum\nolimits_{j=1}^{n} \tilde{c}_{kj} x_j , \qquad k = 1,...,K \\
&\min \quad \tilde{f}_r : \sum\nolimits_{j=1}^{n} \tilde{c}_{rj} x_j , \qquad r = K+1,...,R \\
&\text{subject to}: \\
&\sum\nolimits_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i , \qquad i = 1,...,m \\
&\sum\nolimits_{j=1}^{n} x_j = 1, \\
&x_j \ge 0, \qquad j = 1,...,n,
\end{aligned} \tag{4}
$$

where $\tilde{c}_{kj}$, $\tilde{c}_{rj}$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are random parameters. In the CCP approach, the Program (4) with considering positive objectives is converted into a deterministic equivalent program as follows (Prékopa, 1995):

$$
\begin{aligned}
&\max \quad E\left(\sum\nolimits_{j=1}^{n} \tilde{c}_{kj} x_j\right) , \quad k = 1,...,K \\
&\text{subject to}: \\
&\Pr\left(\sum\nolimits_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i\right) \ge 1-\alpha_i , \quad i = 1,...,m \\
&\sum\nolimits_{j=1}^{n} x_j = 1, \\
&x_j \ge 0, \qquad j = 1,...,n,
\end{aligned} \tag{5}
$$

where $E\left(\sum\nolimits_{j=1}^{n} \tilde{c}_{kj} x_j\right)$ is the expected value of the objectives with regards to random conditions $\tilde{c}_{kj}$ and α_i are threshold values of constraints that are specified by the DM.

3. Chance constrained goal attainment programming

In this section, we propose our deterministic equivalent program for the Program (4) based on CCP approach and GAP method. We call the resulting approach chance constrained goal attainment programming (CCGAP). In the following, we present how to transform the random objectives and the random constraints.

3.1. Random objectives

$\tilde{c}_{kj}$ are random and normally distributed parameters. The goal g_k can be random parameter where the DM does not know its value with certainty. Thus we assume that the goals of stochastic objectives $\tilde{g}_k \approx N(\mu_{\tilde{g}_k}, \sigma^2_{\tilde{g}_k})$ where $\mu_{\tilde{g}_k}$ and $\sigma^2_{\tilde{g}_k}$ (for k = 1, …,K) are known (Aouni et al., 2005). The objective is maximizing $\sum_{j=1}^{n} \tilde{c}_{kj} x_j$ (for $k = 1,\ldots,K$) and then:

$$\sum_{j=1}^{n} \tilde{c}_{kj} x_j \le \tilde{g}_k, \quad k = 1,\ldots,K$$

By considering the GAP method, it can be said which the objective is minimizing y ($y \ge 0$) subject to:

$$P\ \ \sum_{j=1}^{n} \tilde{c}_{kj} x_j + y \ge \tilde{g}_k\ \ \ge 1 - \alpha_k, \quad k = 1,\ldots,K$$

where ${}_k$ is the threshold value of objective k.

$$P\ \ \tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j \le y\ \ \ge 1 - \alpha_k, \quad k = 1,\ldots,K$$

Let $\tilde{h}_k(x) = \tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j$, $\tilde{h}_k(x)$ is normally distributed and let $E(\tilde{h}_k(x))$ and $\mathrm{Var}(\tilde{h}_k(x))$ be respectively the mean and the variance.

$$P\ \ \tilde{h}_k(x) \le y\ \ \ge 1 - \alpha_k, \quad k = 1,\ldots,K$$

and it can be said:

$$P\left(\frac{\tilde{h}_k(x) - E(\tilde{h}_k(x))}{\sqrt{\mathrm{Var}(\tilde{h}_k(x))}} \le \frac{y - E(\tilde{h}_k(x))}{\sqrt{\mathrm{Var}(\tilde{h}_k(x))}} \right) \ge 1 - \alpha_k, \quad k = 1,\ldots,K$$

where $\phi(z) = P\ \ N(0,1) \le z$ represents the probability distribution function of a standard normal distribution.

$$\frac{y - E(\tilde{h}_k(x))}{\sqrt{\mathrm{Var}(\tilde{h}_k(x))}} \ge \phi^{-1}(1 - \alpha_k), \quad k = 1,\ldots,K$$

$$y \ge E(\tilde{h}_k(x)) + \phi^{-1}(1 - \alpha_k)\sqrt{\mathrm{Var}(\tilde{h}_k(x))}, \quad k = 1,\ldots,K$$

$$y \ge E(\tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j) + \phi^{-1}(1 - \alpha_k)\sqrt{\mathrm{Var}(\tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j)}, \quad k = 1,\ldots,K$$

At final it can be said:

$$E(\sum_{j=1}^{n} \tilde{c}_{kj} x_j) - \phi^{-1}(1 - \alpha_k)\sqrt{\mathrm{Var}(\tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j)} + y \ge E(\tilde{g}_k), \quad k = 1,\ldots,K.$$

3.2. Random constraints

Random constraints are handled as in the CCP approach: $\tilde{a}_{ij}$ and $\tilde{b}_i$ are random and normally distributed parameters. The related chance constraint to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i$ is:

$$P\left\{\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i\right\} \ge 1-\alpha_i, \qquad i=1,\dots,m$$

Let $\tilde{l}_i(x) = \sum_{j=1}^{n} \tilde{a}_{ij} x_j - \tilde{b}_i$, $\tilde{l}_i(x)$ is normally distributed and let $E(\tilde{l}_i(x))$ and $\mathrm{Var}(\tilde{l}_i(x))$ be respectively the mean and the variance. Thus we have:

$$P\left\{\tilde{l}_i(x) \le 0\right\} \ge 1-\alpha_i, \qquad i=1,\dots,m$$

and then

$$P\left(\frac{\tilde{l}_i(x) - E(\tilde{l}_i(x))}{\sqrt{\mathrm{Var}(\tilde{l}_i(x))}} \le \frac{-E(\tilde{l}_i(x))}{\sqrt{\mathrm{Var}(\tilde{l}_i(x))}}\right) \ge 1-\alpha_i, \qquad i=1,\dots,m$$

where $\phi(z) = P\left\{N(0,1) \le z\right\}$ represents the probability distribution function of a standard normal distribution.

$$\frac{-E(\tilde{l}_i(x))}{\sqrt{\mathrm{Var}(\tilde{l}_i(x))}} \ge \phi^{-1}(1-\alpha_i), \qquad i=1,\dots,m$$

$$E(\tilde{l}_i(x)) + \phi^{-1}(1-\alpha_i)\sqrt{\mathrm{Var}(\tilde{l}_i(x))} \le 0, \qquad i=1,\dots,m$$

At final it can be said:

$$E\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j\right) + \phi^{-1}(1-\alpha_i)\sqrt{\mathrm{Var}\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j - \tilde{b}_i\right)} \le E(\tilde{b}_i), \qquad i=1,\dots,m.$$

The CCGAP which is equivalent to Program (4) can be formulated as:

$$\begin{aligned}
&\min \quad y\\
&\text{subject to}:\\
&E\left(\sum_{j=1}^{n} \tilde{c}_{kj} x_j\right) - \phi^{-1}(1-\alpha_k)\sqrt{\mathrm{Var}\left(\tilde{g}_k - \sum_{j=1}^{n} \tilde{c}_{kj} x_j\right)} + w_k y \ge E(\tilde{g}_k), \qquad k=1,\dots,K\\
&E\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j\right) + \phi^{-1}(1-\alpha_i)\sqrt{\mathrm{Var}\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j - \tilde{b}_i\right)} - w_i y \le E(\tilde{b}_i), \qquad i=1,\dots,m\\
&\sum_{j=1}^{n} x_j = 1,\ x_j \ge 0,\ y \ge 0, \qquad j=1,\dots,n,
\end{aligned} \tag{6}$$

where $\sum_{k=1}^{K} w_k + \sum_{i=1}^{m} w_i = 1$($w_k$, $w_i > 0$, for $k = 1, \ldots, K$ and $i = 1, \ldots, m$).

The Program (6) allows DM's ideas about random parameters, conflicting objectives and values of threshold be gathered prior to optimization process. In next section, we will illustrate our proposed model for a multi-objective stochastic problem about optimal portfolio selection in Iran stock market.

4. Case study

In this section we will illustrate our developed model through a portfolio selection real problem where the goals of stochastic objectives and rate of return of securities are random parameters. We consider a sample of 15 stocks from the Iran stock exchange market. The data and observations (from March 2002 to March 2011) of the in-sample period are used as the training set to determine the models parameters and specifications. For the illustration purpose, we consider four selected objectives. These objectives are:

● The first objective is return stochastic objective function. The rate of return ($\tilde{r}_j = (\tilde{P}_{j,t} - P_{j,t-1} + \tilde{D}_{j,t}) / P_{j,t-1}$) measures the profitability of the stock where the income can be in the form of random capital gain and dividend. Here $\tilde{P}_{j,t}$ is the price of the stock j at time t and $\tilde{D}_{j,t}$ is the dividend received during the period $[t-1, t]$. $\tilde{r}_j$ $j = 1, \ldots, 15$ are random and normally distributed with known mean μ_j and variance σ^2_j. This stochastic objective is to be maximized.

● The second objective is Beta risk objective function. $\beta_j = \mathrm{Cov}(\tilde{r}_j, \tilde{r}_m) / \mathrm{Var}(\tilde{r}_m)$, where $\tilde{r}_j$, $j = 1, \ldots, 15$ is the rate of return of stock j and $\tilde{r}_m$ is the rate of market return. This objective indicates the reliance of stock's return on market. Lower correlation with the market indicates the stock performance on its own rather than by the movements of the market. The aim is to choose a diversified portfolio with small β.

● The third objective is initial cost of investment objective function. In real world, many people suffer because they have not enough money for secure investments. Thus the aim this is which they spend less money while will obtain their favorite results from other objectives. P_j is the price of stock j (with known formal currency) in the last under study day. Let N be total number of existent securities (stocks) in the optimum portfolio. Therefore the initial cost of investment objective function can be obtained without considering the value N as follows:

$$Z = P_1(Nx_1) + P_2(Nx_2) + \ldots + P_{15}(Nx_{15}) \Rightarrow Z = N(P_1x_1 + P_2x_2 + \ldots + P_{15}x_{15})$$

$$\Rightarrow Z = N\left(\sum_{j=1}^{15} P_j x_j\right) \Rightarrow \frac{Z}{N} = \frac{N\left(\sum_{j=1}^{15} P_j x_j\right)}{N} \Rightarrow \frac{Z}{N} = f_3 = \sum_{j=1}^{15} P_j x_j$$

Finally optimum value of cost for selection and allocation of optimum portfolio is equal to $Z^* = f_3^* N$. We consider price of the last day in under study term (P_j) to purchase stock j. This objective is to be minimized.

● The fourth objective is purchase ratio objective function. $PR_j = PN_{j,s} / PN_T$ (for j = 1, …, 15) is purchase ratio of stock j when the market is open. $PN_{j,s}$ being the number of purchasers of stock j and PN_T being the total number of purchases in under study term. The aim is to select a portfolio whose stocks are more attracting to buyers. This objective has to be maximized.

The system constraints can be defined as follows:

(**a**) The sum of the proportions invested in stocks is equal to 1: $\sum_{j=1}^{15} x_j = 1$ (Markowitz, 1952).

(**b**) Allocated constraints:

● In order to diversify the selected portfolios and maximum utilization from the all existent capacities of investment, DM proposes to invest 25% in automotive industry (for stocks j = 1, 2, 3, 15), banking and leasing (for stocks j = 5, 6, 7, 8), investment sectors (for stocks j = 4, 13, 14) and another sectors (for stocks j = 9, 10, 11, 12). In fact summation of these constraints is equal to the constraint of part (**a**).

● Setting a lower and an upper bound for each stock in order to diversify the portfolio, $0 \le x_j \le 0.1$, for j = 1,..., 15, where the x_j is the proportion to be invested in the stock j.

The main portfolio selection problem can be formulated as follows:

$$
\begin{aligned}
&\max \quad f_1 = \sum_{j=1}^{15} \tilde{r}_j x_j \\
&\min \quad f_2 = \sum_{j=1}^{15} \beta_j x_j \\
&\min \quad f_3 = \sum_{j=1}^{15} P_j x_j \\
&\max \quad f_4 = \sum_{j=1}^{15} PR_j x_j \\
&\text{subject to:} \\
&x_1 + x_2 + x_3 + x_{15} = 0.25, \\
&x_5 + x_6 + x_7 + x_8 = 0.25, \\
&x_4 + x_{13} + x_{14} = 0.25, \\
&x_9 + x_{10} + x_{11} + x_{12} = 0.25, \\
&0 \le x_j \le 0.1, \qquad j = 1, \ldots, 15.
\end{aligned}
\tag{7}
$$

The Program (7) is transformed to a CCGAP as Program (8):

$$
\begin{aligned}
&\min \quad y \\
&\text{subject to:} \\
&E\left(\sum_{j=1}^{15} \tilde{r}_j x_j\right) - \phi^{-1}(1-\alpha)\sqrt{\mathrm{Var}\left(\tilde{g}_1 - \sum_{j=1}^{15} \tilde{r}_j x_j\right)} + w_1 y \ge E(\tilde{g}_1), \\
&\sum_{j=1}^{15} \beta_j x_j - w_2 y \le g_2, \\
&\sum_{j=1}^{15} P_j x_j - w_3 y \le g_3, \\
&\sum_{j=1}^{15} PR_j x_j + w_4 y \ge g_4, \\
&x_1 + x_2 + x_3 + x_{15} = 0.25, \\
&x_5 + x_6 + x_7 + x_8 = 0.25, \\
&x_4 + x_{13} + x_{14} = 0.25, \\
&x_9 + x_{10} + x_{11} + x_{12} = 0.25, \\
&0 \le x_j \le 0.1, y \ge 0, \qquad j = 1, \ldots, 15,
\end{aligned}
\tag{8}
$$

where $\sum_{k=1}^{4} w_k = 1$ ($w_k > 0$, for $k = 1, \ldots, 4$).

In Program (8), the parameters $\tilde{r}_j$ (for $j = 1, \ldots, 15$) and $\tilde{g}_1$ are considered random and normally distributed with known mean and variance. Table 1, presents the mean and variance values of the random parameter $\tilde{g}_1$. Also Table 2 presents the goal values of deterministic objectives. The goal value of Beta objective is equal to 1 (Lee and Chesser, 1980). The other parameters of Program (8) are known with certainty. The objectives considered in this example are the rate of return, the risk β, the initial cost of investment and the purchase ratio which only the return objective is stochastic and other objectives are deterministic.

Table 1. Goal value of stochastic objective (for *k* = 1)

Random goal	μ	σ^2
$\tilde{g}_1$	0.1293075	0.0003256

Table 2. Goal values of deterministic objectives (for *k* = 2, 3, 4)

Objectives (f_k)	Deterministic goals (g_k)
f_2	1
f_3	1262
f_4	0.1353624

In Table 3, we present data concerning the 15 different stocks of the Iran stock market for the years 2002 to 2011. The six columns of the Table 3 are number, the stocks, the stock price in the last exchanged day, the risk β, the expected rate of return of each security, and the purchase ratio of each security, respectively.

From DM's viewpoint, different scenarios are proposed for threshold α and vector of preference weights *w*. Program (8) was applied separately for each scenario by Lingo software package and Table 4 shows optimal portfolio of each scenario and optimal values of each objective.

Table 3. Data under study

j	Stock	Stock price in the last exchanged day (P_j)	Beta risk (β_j)	Expected rate of return (μ_j)	Purchase ratio (PR_j)
1	PARS AUTO	926	0.59815	0.0012654	0.1292097
2	MEH IRAN AUTO	700	1.15065	–0.0006437	0.1301761
3	SAIPA	926	0.17812	0.0015994	0.1214500
4	RAY SAIPA INV	2392	2.60025	0.0027148	0.1067670

5	PERSIAN BANK	2337	1.05606	0.0021114	0.1140610
6	KAR AFR BANK	1435	2.00207	0.0019685	0.1304640
7	IRAN LEAS	2115	–1.02369	0.0027717	0.1304920
8	IND & MIN LEAS	967	1.23007	0.0022249	0.1399780
9	PARS ALU	948	2.14956	–0.0001838	0.1289632
10	ALUMTAK	1385	–0.82301	0.0016264	0.1100235
11	IRAN BEHNUSH	2373	–0.00125	0.0009780	0.1224789
12	PARS MINOO	2477	3.67891	–0.0021901	0.1263525
13	OIL IND INV	1180	1.67921	0.0011433	0.1324790
14	SEPAH INV	1180	2.12003	0.0011433	0.1240011
15	SAIPA DIESEL	920	0.89782	–0.0004956	0.1203698

Table 4. Optimum values of portfolios with regard to difference scenarios

α = 0.01

w	(0.2,0.2,0.4,0.2)	(0.2,0.5,0.1,0.2)	(0.25,0.25,0.25,0.25)	(0.3,0.2,0.2,0.3)
x_1^*	0.04232	0.02658	0.02795	0.02704
x_2^*	0.1	0.1	0.1	0.1
x_3^*	0.1	0.07151	0.08488	0.07597
x_4^*	0.05	0.05	0.05	0.05
x_5^*	0	0	0	0
x_6^*	0.1	0.1	0.1	0.1
x_7^*	0.05	0.05	0.05	0.05
x_8^*	0.1	0.1	0.1	0.1
x_9^*	0.1	0.1	0.1	0.1
x_{10}^*	0.1	0.1	0.1	0.1
x_{11}^*	0.05	0.05	0.05	0.05
x_{12}^*	0	0	0	0
x_{13}^*	0.1	0.1	0.1	0.1
x_{14}^*	0.1	0.1	0.1	0.1
x_{15}^*	0.00768	0.05191	0.03717	0.047
f_1^*	0.0012608	0.0011734	0.00120383	0.00118354
f_2^*	1	1	1	1
f_3^*	1262.3	1262.1	1262.2	1262.1
f_4^*	0.12613298	0.12596307	0.12598964	0.12597193

α = 0.025

w	(0.2,0.2,0.4,0.2)	(0.2,0.5,0.1,0.2)	(0.25,0.25,0.25,0.25)	(0.3,0.2,0.2,0.3)
x_1^*	0.03973	0.025	0.02631	0.02543
x_2^*	0.1	0.1	0.1	0.1

x_3^*	0.1	0.07244	0.08523	0.07671
x_4^*	0.05	0.05	0.05	0.05
x_5^*	0	0	0	0
x_6^*	0.1	0.1	0.1	0.1
x_7^*	0.05	0.05	0.05	0.05
x_8^*	0.1	0.1	0.1	0.1
x_9^*	0.1	0.1	0.1	0.1
x_{10}^*	0.1	0.1	0.1	0.1
x_{11}^*	0.05	0.05	0.05	0.05
x_{12}^*	0	0	0	0
x_{13}^*	0.1	0.1	0.1	0.1
x_{14}^*	0.1	0.1	0.1	0.1
x_{15}^*	0.01027	0.05256	0.03846	0.04786
f_1^*	0.00125625	0.00117257	0.00120167	0.00118227
f_2^*	1	1	1	1
f_3^*	1262.3	1262.1	1262.2	1262.1
f_4^*	0.12611010	0.12595007	0.12597549	0.12595854
		α = 0.05		
w	(0.2,0.2,0.4,0.2)	(0.2,0.5,0.1,0.2)	(0.25,0.25,0.25,0.25)	(0.3,0.2,0.2,0.3)
x_1^*	0.03753	0.02311	0.02437	0.02353
x_2^*	0.1	0.1	0.1	0.1
x_3^*	0.1	0.07378	0.08607	0.07788
x_4^*	0.05	0.05	0.05	0.05
x_5^*	0	0	0	0
x_6^*	0.1	0.1	0.1	0.1
x_7^*	0.05	0.05	0.05	0.05
x_8^*	0.1	0.1	0.1	0.1
x_9^*	0.1	0.1	0.1	0.1
x_{10}^*	0.1	0.1	0.1	0.1
x_{11}^*	0.05	0.05	0.05	0.05
x_{12}^*	0	0	0	0
x_{13}^*	0.1	0.1	0.1	0.1
x_{14}^*	0.1	0.1	0.1	0.1
x_{15}^*	0.01247	0.05311	0.03956	0.04859
f_1^*	0.00125237	0.00117205	0.00120001	0.00118137
f_2^*	1	1	1	1
f_3^*	1262.3	1262.1	1262.2	1262.1

f_4^*	0.12609063	0.12593482	0.12595924	0.12594296
		α = 0.1		
w	(0.2,0.2,0.4,0.2)	(0.2,0.5,0.1,0.2)	(0.25,0.25,0.25,0.25)	(0.3,0.2,0.2,0.3)
x_1^*	0.03498	0.01978	0.02099	0.02018
x_2^*	0.1	0.1	0.1	0.1
x_3^*	0.1	0.07647	0.08818	0.08037
x_4^*	0.05	0.05	0.05	0.05
x_5^*	0	0	0	0
x_6^*	0.1	0.1	0.1	0.1
x_7^*	0.05	0.05	0.05	0.05
x_8^*	0.1	0.1	0.1	0.1
x_9^*	0.1	0.1	0.1	0.1
x_{10}^*	0.1	0.1	0.1	0.1
x_{11}^*	0.05	0.05	0.05	0.05
x_{12}^*	0	0	0	0
x_{13}^*	0.1	0.1	0.1	0.1
x_{14}^*	0.1	0.1	0.1	0.1
x_{15}^*	0.01502	0.05375	0.04084	0.04944
f_1^*	0.00124787	0.00117182	0.00119847	0.00118070
f_2^*	1	1	1	1
f_3^*	1262.3	1262.1	1262.2	1262.1
f_4^*	0.12606806	0.12590835	0.12593163	0.12591611

The obtained results from different scenarios in Table 4 show some kind close trade-offs between objectives. We consider that $\tilde{r}_j$ (for j = 1,…, 15) and $\tilde{g}_1$ are normally distributed with known mean and variance. In Table 4 the all portfolios were obtained with considering uncertainty. We notice that the results in all portfolios are close to each other, where only the proportions invested in some stocks are changed.

What is understood from Table 4 is that by increasing α in each vector w, f_1^* will decrease. In other words increase of α can result in worst condition of access to expected rate of return under uncertainty. About f_2^*, it should be said that investment risk in any level of uncertainty is equal to 1. The results indicate that changes α in each vector w have not influence on f_3^*, so that under uncertainty and at each level α, more amount from the allocated budget is needed. By increase of α in each vector w, f_4^* will have a decreasing procedure. In the other words, by increase of acceptance probability of uncertainty, number of purchasers will decrease. It seems investment in portfolio under uncertainty has less attractiveness for purchasers and investors. At final, it seems that increase of α in each scenario w has undesired effects on problem's objectives and deterioration procedure of objectives can be seen. Of course this deterioration procedure of optimal values of objectives is evident and rational, because it provides a better and more tangible represent of reality of investment in Iran stock market under uncertainty for DM.

5. Conclusion

Investment in multi-objective portfolios is important from two aspects of level of use of DM's ideas and considering all or some of random parameters. In the beginning of this paper, the GAP method was introduced as a method in priori category and advantages of this method compared to the GP method was presented. In the other section, the final shape of stochastic programming problems and their optimization by CCP approach were introduced, so that by combining CCP approach and GAP method, we proposed the CCGAP method which can optimize multi-objective stochastic problems. At the end of this paper, on the basis of different scenarios, the proposed model was applied for a real problem of multi-objective portfolio selection by four objectives in Iran stock exchange market. In this problem, the rate of return of securities and the goal value of return objective were considered as random parameters with normal distribution and known mean and variance. At final, the obtained results from optimization of stochastic problem of multi-objective portfolio selection by the CCGAP method indicated that by increase of threshold value in each scenario, objectives improvement of stochastic problem will decrease.

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REFERENCES

[1] **Amiri, M., Ekhtiari, M., Yazdani, M. (2011),** ***Nadir Compromise Programming: A Model for Optimization of Multi-objective Portfolio Problem*** . *Expert Systems with Applications*, 38, 7222–7226;

[2] **Andersson, J. (2000),** ***A Survey of Multi-objective Optimization in Engineering Design***. Department of Mechanical Engineering, Linköping University, Sweden;

[3] **Aouni, B., Abdelaziz, F.B., Martel, J.M. (2005),** ***Decision-maker's Preferences Modeling in the Stochastic Goal Programming***. *European Journal of Operational Research*, 162, 610–618;

[4] **Ballestero, E. (2001),** ***Theory and Methodology, Stochastic Goal Programming: A Mean- variance Approach***. *European Journal of Operational Research,* 131, 476–481;

[5] **Ballestero, E. (2005),** ***Mean-semi Variance Efficient Frontier: A Downside Risk Model for Portfolio Selection***. *Applied Mathematical Finance*, 12, 1–15;

[6] **Bell, D.E., Raiffa, H., Tversky, A. (1988),** ***Risky Choice Revisited***. In: *D.E. Bell* (Ed.), *Decision Making: Descriptive, normative and prescriptive interactions (pp. 99–112). Cambridge: Cambridge University Press;*

[7] **Canakoglu, E., Ozekici, S. (2009),** ***Portfolio Selection in Stochastic Markets with Exponential Utility Functions***. *Annals of Operations Research*, 166, 281-297;

[8] **Charnes, A., Cooper, W.W., Ferguson, R.O. (1955),** ***Optimal Estimation of Executive Compensation by Linear Programming*** . *Management Science,* 1, 138–151;

[9] **Charnes, A., Cooper, W.W. (1963),** ***Deterministic Equivalents for Optimizing and Satisfying under Chance Constraints***. *Operations Research,* 11, 18–39;

[10] **Elton, E.J., Gruber, M.J., Padberg, M. (1976) ,** ***Simple Rules for Optimal Portfolio Selection***. *Journal of Finance*, 31, 1341–1357;

[11] **Gembicki, F.W. (1974),** ***Vector Optimization for Control with Performance and Parameter Sensitivity Indices*** . *PhD. Dissertation, Case Western Reserve University, Cleveland, Ohio*;

[12] **Gembicki, F.W., Haimes, Y.Y. (1975),** ***Approach to Performance and Sensitivity Multi-objective Optimization: The Goal Attainment Method*** . *IEEE Transaction on Automatic Control*, 20, 769–771;

[13] **Georgescu, V. (2011).** ***An Econometric insight Predicting Bucharest Stock Exchange Mean-return and Volatility-return Processes***. *Economic Computation and Economic Cybernetics Studies and Research,* 45, *ASE Publishing;* Special section p1;

[14] **Konno, H. (1990),** ***Piecewise Linear Risk Functions and Portfolio Optimization***. *Journal of the Operations Research Society of Japan*, 33, 139–156;

[15] **Lee, S.M., Chesser, D.L. (1980),** ***Goal Programming for Portfolio Selection***. *Journal of Portfolio Management*, 6, 22–26;

[16] **Liu, L. (1999),** ***Approximate Portfolio Analysis*** . *European Journal of Operational Research*, 119, 35–49;

[17] **Markowitz, H. (1952),** ***Portfolio selection***. *The Journal of Finance,* 7, 77–91;

[18] **Muhlemann, A.P., Lockett, A.G., Gear, A.E. (1978),** ***Portfolio Modeling in Multiple-criteria Situations under Uncertainty.*** *Decision Sciences*, 9, 612–626;

[19] **Prékopa, A.(1995), *Stochastic Programming* .** *London: Kluwer Academic Publishers*;

[20] **Serban, F., Stefanescu, M.V., Ferrara. M. (2011), *Portfolio Optimization and Building of its Efficient Frontier*** . *Economic Computation and Economic Cybernetics Studies and Research*, 45, *ASE Publishing*, 125–137;

[21] **Sharpe, W.F. (1967), *A Linear Programming Algorithm for Mutual Fund Portfolio Selection***. *Management Science*, 13, 499–510;

[22] **Shing, C., Nagasawa, H. (1999), *Interactive Decision System in Stochastic Multi-objective Portfolio Selection*** . *International Journal of Production Economics*, 60–61, 187–193;

[23] **Taha, H.A. (1976), *Operations Research: An Introduction*** ; *Second Edition, New York: Macmillan Publishing Co, Inc*.;

[24] **Tamiz, M., Hasham, R., Jones, D.F., Hesni, B., Fargher, E.K. (1996), *A Two Staged Goal Programming Model for Portfolio Selection*** . In: *Tamiz, M*., (Ed.), Lecture Notes in Economics and Mathematical Systems 432, 286–299;

[25] **Tang, W., Han, Q., Li, G. (2001), *The Portfolio Selection Problems with Chance-Constrained* .** *IEEE Systems Man and Cybernetics*, 4, 2674-2679;

[26] **Xu, H., Zhang, D. (2012), *Monte Carlo Methods for Mean-risk Optimization and Portfolio Selection***. *Computational Management Sciences*, 9, 3-29;

[27] **Young, M. (1998), *A Minimax Portfolio Selection Rule with Linear Programming Solution*** . *Management Science*, 44, 673–683.